Indian J Phys (May 2016) 90(5):583–588
DOI 10.1007/s12648-015-0781-x

REVIEW PAPER

Casimir energy between a sinusoidally corrugated sphere and a plate using proximity force approximation

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Received: 03 March 2015 / Accepted: 28 July 2015 / Published online: 7 October 2015

Abstract: The aim of this paper is to obtain the Casimir energy between a sinusoidally corrugated sphere and a plate. We first present a derivation of the sphere-plate Casimir force obtained by applying proximity force approximation, and afterwards, we obtain the effect of deformation on the Casimir interaction energy by considering the sphere to have different kinds of sinusoidal corrugations. We suppose $a/r \leq 0.00755$ for the sake of proximity force approximation validity and an experimental accuracy goal of 1 % in the case of a sphere with the radius r at a minimal distance a from a plate. The effect of finite conductivity is taken into account for such short distances. Furthermore, we consider that mirrors are not perfect reflectors indeed which concludes in exact theoretical prediction. Thus, we consider the correction due to finite conductivity. We note that for open geometries, temperature dependence of the Casimir force is larger than that of closed geometries. We also investigate thermal corrections of the Casimir force.

Keywords: Casimir energy; Corrugated sphere; Proximity force approximation

PACS Nos.: 03.70.+k; 11.10.−z; 31.30.jh

1. Introduction

The Casimir effect is the dominant interaction between two neutral plates with micron or submicron range separation distance [1]. This tremendous effect with macroscopic manifestation has got a significant role in miniaturized physical systems. This quantum effect originated from modification of the vacuum fluctuations has been investigated for a variety of fields, geometries, number of spatial dimensions and boundary conditions [2, 3]. A large number of experiments are done to verify the realization of the Casimir effect [4–8]. Considering that maintaining parallel plates has been a challenge experimentally, most of the experiments are done using the configuration of sphere-plate instead of two parallel plates geometry [5, 9, 10].

According to the issue that the sphere-plate geometry is a highly relevant experimental configuration, it faces rapid theoretical and experimental advancement. Some of the studies containing the sphere-plate electromagnetic vacuum energy have been extremely done since 2007 by many researchers [11–14]. They have performed exact Casimir calculations for sphere-plate geometries based on the scattering matrix methods as a proficient technique to study the interaction energy, because in this approach the accuracy of comparison between theory and experiment can be determined by the levels of precision. In these investigations, the plate is supposed to be perfectly conducting, while the electromagnetic field on the sphere is satisfied with either a perfect conductor boundary condition or an imperfect conductor boundary condition. The repulsion in the sphere-plate geometry has been studied in [15], using the approaches reported in [11–14] with the aim of finding the constraints on the repulsion caused by magnetic permeability of the sphere. Performing analytical and numerical worldline approaches for the Casimir effect of the sphere-plate and cylinder-plate geometries [16], the interplay between these geometries and temperature has been investigated thoroughly. Applying the functional determinant method, the finite temperature Casimir effect for a sphere and a plate—as a geometry of experimental interest—has been investigated focusing on the limiting cases [17]. Chen et al. [18] are the first to calculate and measure the lateral Casimir force between sinusoidally corrugated plate and sphere (in the case of real metals of

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finite conductivity) based on the atomic force microscope. Chiu et al. [19] have presented the measurement data of the lateral Casimir force between two corrugated surfaces of sphere and plate in the nonadditive regime. They have performed a complete comparison of these measurement data with the exact theory and the predictions of PFA [20]. Even the non-equilibrium Casimir force of the sphere-plate geometry has been treated both analytically and numerically [21]. Calculations of the Casimir force between a plate and a nanostructured surface at finite temperature has been presented in the framework of the scattering theory [22]. The Casimir–Polder forces between an atom and a surface of arbitrary uniaxial corrugations have been presented applying a fully nonperturbative technique in the height profile of the corrugation [23]. Kruger et al. [24] have recently investigated the effect of roughness or surface modulations on the distance dependence power law of the interactions between curved objects at proximity using height distribution functions. Applying the worldline numerics [25], Gies et al. have examined the Casimir interaction energies for the sphere-plate and cylinder-plate configuration due to the scalar-field fluctuations under the Dirichlet boundary conditions for a wide range of curvature parameters. Based on a high-precision calculation and using these worldline numerics, they have determined the validity bounds of the proximity force approximation quantitatively. They have observed that for the accuracy goal of 1 %, the PFA is valid for $a/r \leq 0.00755$, where a/r is called the curvature parameter, r is the radius of the sphere at a minimal distance a from a plate [26]. Devoted to study the force between an infinite plane surface and an arbitrary curved one, the validity of PFA in the weak coupling is presented for all temperatures by Milton et al. [27] who have also obtained exact results for the Casimir energy between two weak-coupled semitransparent spheres at arbitrary temperatures.

Analysis shows that using PFA for obtaining plate-sphere result from the plate–plate conclusion for corrugations with wave length λ is a good approximation as long as ra is much bigger than λ^2 and the amplitude of corrugation is smaller than other length scales [28]. Application of the proximity force approximation for corrugated surfaces may not obviously lead to precise expressions. However, use of this approximation conducts in a simple procedure for achieving the purpose and emitting the difficulties appearing in the exact investigations.

Because of the experimental importance of the sphere-plate configuration as well as the strong dependence of the Casimir force on the interaction bodies' properties, their surface states, their shapes and many other factors, this paper is devoted to calculation of the Casimir force between a sinusoidally corrugated sphere and a plate. We try to obtain the Casimir energy for the highly relevant experimental configuration of sphere-plate geometry for different kinds of corrugations on the sphere based on PFA as well as thermal and finite conductivity corrections.

2. The Casimir interaction energy for sphere-plate geometry

The approximate Casimir energy between two curved surfaces placed at a short separation distance can be obtained as a sum of energies between a pair of small parallel plates [27–30]. Considering this approximation, we try to use two parallel plates' Casimir energy to obtain the Casimir force for the sphere-plate geometry. The electromagnetic Casimir energy density for two parallel perfect conductor plates with separation distance H is given by [2, 31, 32]

$$\varepsilon_{pp}(H) = -\frac{\pi^2}{720H^3}, \tag{1}$$

and consequently, the normal Casimir force density for this configuration is

$$F_{pp}(H) = -\frac{\pi^2}{240H^4}. \tag{2}$$

Let us consider a sphere with radius r above a plate, located in the shortest distance a. The Casimir force between the sphere and the plate can be obtained by integrating the Casimir force density over the half-sphere which is opposite to the plate

$$F_{sp} = \int F_{pp}(H)\mathrm{d}s, \tag{3}$$

where the subscript sp stands for the sphere-plate geometry, similar to cylinder-plate geometry [33]. One can use polar coordinates to determine the normal distance of a point of the sphere with parameter θ, from the plate as $H = a + r(1 - \cos\theta)$. Thus, the Casimir force for this geometry can be written as

$$F_{sp} = -\frac{\pi^3 r^2}{120}\int_0^{\pi/2}\frac{\sin\theta}{[a + r(1-\cos\theta)]^4}\mathrm{d}\theta, \tag{4}$$

and considering that $a \prec\prec r$, the Casimir interaction force reads

$$F_{sp} = -\frac{\pi^3 r}{360a^3}. \tag{5}$$

Integrating the normal Casimir force with respect to the separation distance a, the Casimir interaction energy between a sphere and a plate with no corrugations is given by

$$E_{\mathrm{sp}} = \int_a^\infty F_{\mathrm{sp}} \mathrm{d}a = -\frac{\pi^3 r}{720 a^2}. \tag{6}$$

It is worth mentioning that in addition to the geometry, other parameters such as finite temperature, finite conductivity and surface roughness must be considered. In the experiments, the effect of imperfect reflections of mirrors on the Casimir force is noticeable; therefore, it is necessary to consider finite conductivity to obtain precise theoretical predictions [34] Lambrecht and Marachevsky [35] have obtained the exact Casimir force between two arbitrary periodic dielectric gratings. They have found that considering finite conductivity gives a smaller deviation of the exact force from the PFA prediction than the calculation of perfect mirrors. Considering the finite conductivity of metals, the corresponding corrections may be included in the mentioned Casimir energy per unit area for two parallel perfect conductor plates [19, 36–38] as

$$\varepsilon_{pp}(H) = -\frac{\pi^2}{720 H^3}\left(1 + \sum_{n=1}^{4} C_n \left(\frac{\lambda_p}{2\pi H}\right)^n\right) \tag{7}$$

where λ_p is the plasma wavelength and the coefficients C_n are

$$C_1 = -4, C_2 = \frac{72}{5}, \quad C_3 = -\frac{320}{7}\left(1 - \frac{\pi^2}{210}\right),$$
$$C_4 = \frac{400}{3}\left(1 - \frac{163\pi^2}{7350}\right).$$

Equation (7) is applicable to the separations $H \geq \lambda_p$. Imposing PFA approximation on the correction terms leads to the following correction in the force derived by Eq. (5)

$$\Delta F_{\mathrm{sp}} \approx -\frac{\pi^3 r}{120} \sum_{n=1}^{4} C_n \left(\frac{\lambda_p}{2\pi}\right)^n \frac{1}{a^{n+3}}, \tag{8}$$

and integrating the correction of the Casimir force with respect to the separation distance, the corresponding correction to the Casimir energy due to the finite conductivity can be obtained

$$\Delta E_{\mathrm{sp}} \approx -\frac{\pi^3 r}{120} \sum_{n=1}^{4} C_n \left(\frac{\lambda_p}{2\pi}\right)^n \frac{1}{(n+2) a^{n+2}}. \tag{9}$$

3. Results and discussion

3.1. Imposing the effect of deformation on the sphere-plate Casimir interaction energy

The Casimir interaction strongly depends on the shape and orientation of the surfaces; therefore, any kind of deformation in the interacting surfaces may lead to a modification in this interaction. We want to develop PFA for a sphere with periodic corrugations above a smooth plate. This description is valid only to study deformations with large wavelengths. Hence, the roughness effect is obtained as a small correction to the Casimir energy [39–41]

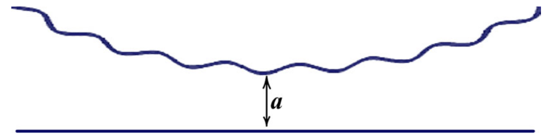

Fig. 1 A cross section of azimuthal corrugated sphere above the plate with $a \prec\prec r$ for PFA applicability in which r is the radius of the sphere and a is the shortest separation distance

At this point, we concentrate on the sinusoidal corrugations. As the first example, assume the sphere to be corrugated in the azimuthal direction as shown in Fig. 1. The following function describes the radius of the corrugated sphere

$$r' = r + A \sin(\mu\varphi) \tag{10}$$

where r' and r are radii of sphere with and without corrugation, respectively, A is the corrugation amplitude and μ is the corrugation's frequency, which is necessarily a positive integer. Considering the additive summation of the Casimir energy of the sphere-plate geometry with no corrugation obtained in Eq. (6) and knowing that the shortest distance between the corrugated sphere and the plate is still a, the Casimir energy for the corrugated sphere (with large corrugation wavelength) and the plate can be obtained as

$$\begin{aligned} E_{\mathrm{sp}}^{\mathrm{cor}} &= \frac{1}{2\pi}\int_0^{2\pi} E_{\mathrm{sp}}(\varphi)\mathrm{d}\varphi \\ &= -\frac{\pi^3}{720a^2}\frac{1}{2\pi}\int_0^{2\pi}(r + A\sin(\mu\varphi))\mathrm{d}\varphi \\ &= -\frac{\pi^2}{720a^2}\left(\pi r\mu + A\sin^2(\mu\pi)\right)/\mu \end{aligned} \tag{11}$$

in which we use the fact that all the radii corresponding to different azimuthal angles introduced in Eq. (10) have equal probability [33, 42–44]

As mentioned before, μ is an integer and therefore the effect of azimuthal corrugation cancelled in Eq. (11). Indeed, the Casimir energy takes the form of sphere-plate energy with no corrugation (Eq. (6)).

In this case of corrugation, the correction of finite conductivity of Eq. (9) yields

$$\Delta E_{\mathrm{sp}}^{\mathrm{cor}} = -\frac{\pi^3}{120}\left(r + \frac{A\sin^2(\mu\pi)}{\mu\pi}\right)\sum_{n=1}^{4} C_n\left(\frac{\lambda_p}{2\pi}\right)^n \frac{1}{(n+2)a^{n+2}} \tag{12}$$

when μ is integer, Eq. (12) returns to Eq. (9) and therefore the effect of the azimuthal corrugation in the finite conductivity correction is emitted.

As another simple example of sinusoidal corrugations, we describe a polar corrugation as

$$r' = r + A\sin(\nu\theta) \tag{13}$$

where ν is the frequency of the corrugation and it must be a positive integer. The Casimir energy corresponding to this configuration is

$$E_{\mathrm{sp}}^{\mathrm{cor}} = \frac{2}{\pi}\int_0^{\pi/2} E_{\mathrm{sp}}(\theta)\mathrm{d}\theta = -\frac{\pi^2}{360a^2}\left[\frac{\pi r}{2} + \frac{2A\sin^2(\pi\nu/4)}{\nu}\right] \tag{14}$$

As illustrated in Fig. 2, for asymptotic large values of ν, the Casimir energy corresponding to the polar corrugation approaches the energy of the case with no corrugation.

The correction due to the finite conductivity with a similar argument leads to the following result

$$\Delta E_{\mathrm{sp}}^{\mathrm{cor}} \approx -\frac{\pi^2}{60}\left[\frac{\pi r}{2} + \frac{2A\sin^2(\pi\nu/4)}{\nu}\right] \sum_{n=1}^{4} C_n \left(\frac{\lambda_p}{2\pi}\right)^n \frac{1}{(n+2)a^{n+2}}. \tag{15}$$

The same as the previous diagram, the energy correction corresponding to the finite conductivity for polar corrugation disappears for asymptotic large ν s.

As the final example, we perform the calculations for one more complete case: a sphere with both azimuthal and polar corrugations. A sphere with sinusoidal corrugation is in some way similar to a golf ball and one may describe this corrugation by the following ansatz

$$r' = r + A\sin(\nu\theta)\sin(\mu\varphi) \tag{16}$$

Even in this case, the shortest distance is still a which is measured from the pole of the sphere (where $\theta, \varphi = 0$) to the plate. The Casimir energy associated with this configuration can be written as

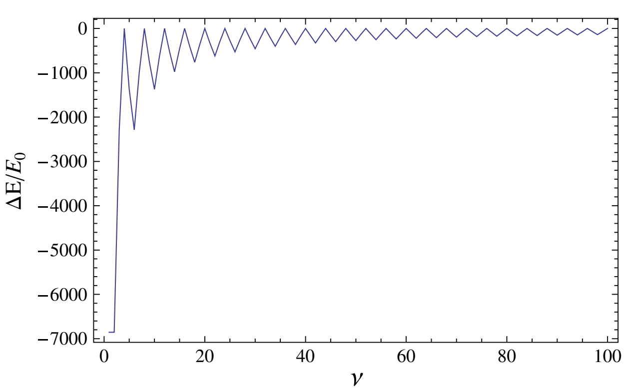

Fig. 2 Correction of the Casimir energy $\Delta E = E_{sp}^{cor}(r) - E_{sp}(r)$ scaled using $E_0 = 3.17 \times 10^{-26} j$ versus the number of the corrugation ν for $A = 4$ pm and $r = 1$ μm

$$\Delta E_{\mathrm{sp}}^{\mathrm{cor}} \approx -\frac{\pi}{720a^2}\left[\pi^2 r + \frac{4A\sin^2(\pi\mu)\sin^2(\pi\nu/4)}{\mu\nu}\right] \tag{17}$$

Considering that for integer μ, $\sin(\pi\mu) = 0$, and this equation takes the form of Eq. (6). This kind of corrugations contributes neither to the Casimir energy of the sphere-plate geometry nor to the correction related to the finite conductivity. Therefore, the finite conductivity correction is still the one mentioned in Eq. (9).

3.2. Finite temperature corrections of the Casimir energy

For two parallel plates, the finite temperature Casimir force per unit area reads [45]

$$F_{\mathrm{pp}}(H) = -\frac{\pi^2}{240H^4} - \frac{\pi^2 T^4}{45} + \frac{\pi T}{H^3}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\frac{k^2}{l}\exp\left(-\frac{\pi k l}{HT}\right) \tag{18}$$

or

$$F_{\mathrm{pp}}(H) = -\frac{\xi_R(H)T}{4\pi H^3} - \frac{2T}{\pi}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{2\pi^2 l^2 T^2}{kH} - \frac{\pi l T}{k^2 H^2} + \frac{1}{4k^3H^3}\right)e^{-4\pi HTkl} \tag{19}$$

with H as the separation distance between the plates.

Equation (18) explains that in the low-temperature region, the thermal correction is dominated by the zero temperature term, and therefore, the leading term of the thermal correction in the low-temperature limit is

$$\Delta F_{\mathrm{pp}}^{\mathrm{Low}\ T} \approx -\frac{\pi^2 T^4}{45}. \tag{20}$$

Imposition of PFA on this term results in the following thermal correction of the normal Casimir force

$$\Delta F_{\mathrm{sp}}^{\mathrm{Low}\ T} \approx -\frac{2\pi^3 T^4 r^2}{45} + \cdots \tag{21}$$

which is independent of a. Consequently, we are capable of obtaining the thermal correction of the Casimir energy. Taking azimuthal corrugation into account, one obtains the following expression for the thermal correction of the Casimir energy as

$$\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T} \approx -\frac{2\pi^3 T^4 a}{45}\frac{1}{2\pi}\int_0^{2\pi}(r + A\sin(\mu\varphi))^2\mathrm{d}\varphi + \cdots = \frac{\pi^3 T^4 a}{45}\left(A^2 + 2r^2\right) + \cdots \tag{22}$$

For the investigated polar corrugation, we obtain

$$\Delta E_{sp}^{\mathrm{Low}\quad T} \approx -\frac{\pi^2 T^4 a}{45}\left[\frac{8Ar\,(1-\cos(\pi v/2)) + A^2\pi v + 2\pi r^2 v}{v}\right] + \cdots \tag{23}$$

and for the golf-ball-like corrugation, the thermal correction of the Casimir energy in the low-temperature region can be obtained in a similar manner.

We introduce

$$\rho^{\mathrm{Low}\ T} \approx \frac{\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T}(r') - \Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T}(r)}{\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T}(r)} \tag{24}$$

where $\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T}(r')$ and $\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ \mathrm{T}}(r)$ are the low-temperature corrections of the Casimir energy in the presence and absence of the corrugations, respectively. We have presented in Fig. 3 that the Casimir energy correction in the low-temperature limit asymptotically approaches zero.

According to Eq. (19), in the high-temperature region the exponential term goes to zero quickly, and therefore, the classical term becomes the first term of the thermal correction

$$\Delta F_{\mathrm{pp}}^{\mathrm{High}\ \mathrm{T}}(H) \approx -\frac{\xi_R(3)T}{4\pi H^3} + \cdots \tag{25}$$

Imposition of the proximity force approximation on Eq. (24) with respect to $r \succ\succ a$ results in

$$\Delta F_{\mathrm{sp}}^{\mathrm{High}\ T} \approx -\frac{\xi_R(3)Tr}{4a^2} + \cdots \tag{26}$$

Performing integral over this correction, one can obtain the energy correction easily. In the case of azimuthal corrugation at high temperatures, the thermal correction of the Casimir energy is

$$\Delta E_{\mathrm{sp}}^{\mathrm{High}\ T} \approx -\frac{\xi_R(3)T}{4a}\left(r + \frac{A\sin^2(\mu\pi)}{\mu\pi}\right) + \cdots \tag{27}$$

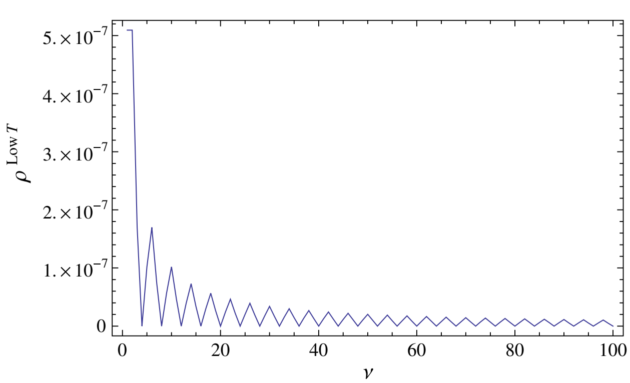

Fig. 3 Plot of the dimensionless parameter ρ^{LowT} versus the number of the corrugation v for $A = 4\ pm$ and $r = 1\ \mu m$

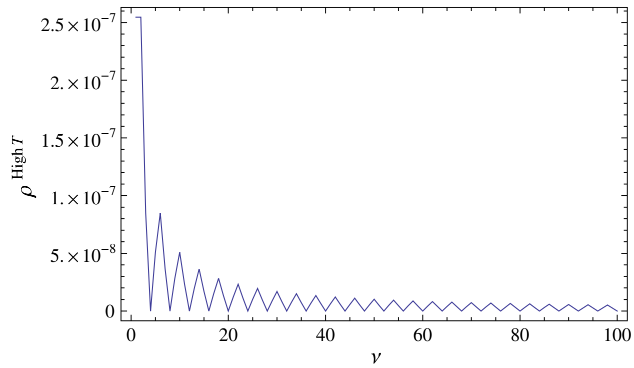

Fig. 4 Calculated dimensionless parameter ρ^{HighT} versus the number of the corrugation v for $A = 4\ pm$ and $r = 1\ \mu m$

Considering that μ is integer, this kind of corrugation does not contribute in the high-temperature correction of the Casimir energy. In the case of polar corrugation, one may obtain

$$\Delta E_{\mathrm{sp}}^{\mathrm{High}\ T} \approx -\frac{\xi_R(3)T}{2a}\left(\frac{r}{2} + \frac{2A\sin^2(v\pi/4)}{v\pi}\right) + \cdots \tag{28}$$

We introduce

$$\rho^{\mathrm{High}\ T} \approx \frac{\Delta E_{\mathrm{sp}}^{\mathrm{High}\ T}(r') - \Delta E_{sp}^{\mathrm{High}\ T}(r)}{\Delta E_{\mathrm{sp}}^{\mathrm{High}\ T}(r)} \tag{29}$$

in which $\Delta E_{\mathrm{sp}}^{\mathrm{Low}\ T}(r')$ and $\Delta E_{\mathrm{sp}}^{\mathrm{High}\ \mathrm{T}}(r)$ are the high-temperature corrections of the Casimir energy in the presence and absence of the corrugations, respectively. Figure 4 shows that this thermal correction of the Casimir effect is significant for small values of v, while it vanishes in the asymptotically large values of v which is the same as what the previous diagrams present.

Eventually in the case of a golf-ball-like corrugation, the correction of the Casimir energy is

$$\Delta E_{\mathrm{sp}}^{\mathrm{High}\ T} \approx -\frac{\xi_R(3)T}{4a}\left(r + \frac{4A\sin^2(\mu\pi)\sin^2(v\pi/4)}{\mu v\pi}\right) + \cdots \tag{30}$$

Considering μ as an integer, Eq. (28) indicates that this kind of corrugation does not contribute in the high-temperature correction of the Casimir energy.

This is interesting enough to note that for large values of separation distance *a,* thermal corrections of the Casimir energy in the high-temperature limit tend to zero, while increasing the same factor in the low-temperature limit concludes in growing the corrections' amount.

4. Conclusions

This paper is devoted to obtaining the Casimir interaction energy between a sinusoidally corrugated sphere and a plate. First, we have presented a derivation for the Casimir force achieved by the proximity force approximation. Application of this procedure has helped us to emit the difficulties appearing in the scattering matrix method, whereas the proximity force approximation does not lead to the exact results. This approximation is applicable for $a/r \leq 0.00755$ and an experimental accuracy goal of 1 % for a sphere with radius r in the surface-to-surface closest distance a from a flat plate. For such short distances, the effect of finite conductivity is not negligible. Furthermore, the effect of imperfect reflections is experimentally noticeable so that considering the finite conductivity, we have obtained the corresponding corrections in the Casimir interaction energy, which has a strong dependence on fields, geometries, number of spatial dimensions and boundary conditions in which any kind of deformation may conclude in a modification. In an attempt along these lines, we have supposed the sphere to be corrugated, and with the aid of the additive summation, we have investigated the effect of deformation. We have considered ra much bigger than λ^2 and the amplitude of the corrugation smaller than other length scales for PFA validity. Due to the fact that the Casimir effect dependence on the temperature in the open geometries such as sphere-plate geometry is significant and experimentally important, we have also considered the thermal corrections of the Casimir force. It is worth mentioning that for large values of separation distance, thermal corrections of the Casimir energy in the high-temperature limit tend to zero, while increasing the same factor in the low-temperature limit concludes in growing the corrections' amount.

Acknowledgments We are grateful to thank A. Moradian for helpful discussion. We would also like to thank B. Döbrich, M. Krüger and H. Gies for helpful suggestions.

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